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On Cluster-Based Channel Identification

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Abstract

In recent years, there has been an increasing interest in applying cluster analysis to channel identification and signal detection. The basic principle underlying this kind of approach is that the cluster information is directly related to the channel impulse response, and therefore it is possible to learn the channel impulse response through cluster analysis. However, a problem that has been long neglected is whether or not the cluster information is sufficient for uniquely identifying the channel. This paper aims to provide an answer to this question.

Keywords: Channel estimation; Channel equalization; Cluster-based channel identircation

1. Introduction

The past decades have witnessed the emergence and development of various approaches to channel identification and equalization, in line with the rapid growth of wideband data communications [1-5]. Among them, the cluster-based approach has attracted an increasing interest for its capability of providing a good balance between computational complexity and performance [1, 2]. In cluster-based approach, the received signal is treated as a mixture of clusters. Channel identification can then be achieved through a two-step procedure: estimating the centers of clusters, and reconstructing the correspondence system between the clusters and the channel states [1, 4].

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This paper is concerned with the blind identifiability based on cluster information, which has been long neglected in the previous studies. For simplicity, BPSK modulation is assumed throughout the paper. However, the conclusions derived herein can be easily extended to other modulation systems.

2. Definitions

Let h ≡[hi]T, i=0, 1, …, L-1, denote the equivalent baseband channel impulse response, where the superscript 'T' denotes matrix transposition and L represents the number of resolvable multipaths. The baseband received signal can then be expressed as follows

$$r(k) = \sum_{i=0}^{L-1} h_i a(k-i) + n(k) \tag{1}$$

where a is the transmitted symbol, and n is the additive white Gaussian noise. In BPSK systems, h and n are real functions, and a(.) is either 1 or -1, corresponding to source symbol s(.) equal to 0 or 1 respectively. Accordingly, the signal component

$$x(k) = \sum_{i=0}^{L-1} h_i a(k-i) \tag{2}$$

has 2L possible values, denoted as xi, each value corresponding to a specific state of L-symbol sequence s(k) ≡ [s(k), s(k-1), …, s(k-L+1)]. In this paper, these values are referred to as origins and are labeled in descending order, i.e., $x_0 \geq x_1 \geq \cdots \geq x_{2^L-1}$, and the states of s(k) are labeled with respect to their distances to all-zero state s0 ≡[0, …, 0, 0], i.e., s1≡ [0, …, 0, 1], s2≡ [0, …, 1, 0], and so forth.

3. Identifiability

Let us begin with an investigation of the rationality of state sequence. Each state sequence can be interpreted as a scheme of origin arrangement. However, all these schemes are infeasible, [s1, s0, s2, s3] for example. A state sequence is said to be rational if it defines a feasible way of origin arrangement. The following theorem gives the necessary conditions for a state sequence to be rational.

Theorem 1. An ordered state sequence of length 2L is rational only if ∀i∈{0, …, 2L-1}, l∈{0, …, L-1}, and j, k∈ Ωl,

Ωl = {0, … 2l-1} + 2l+1 {0, …, 2L-l-1-1},

both of the following equations

$$\phi(i) + \phi(2^L - 1 - i) = 2^L - 1 \tag{3}$$

$$\mathrm{sign}[\phi(j) - \phi(2^l + j)] = \mathrm{sign}[\phi(k) - \phi(2^l + k)] \tag{4}$$

can be satisfied, where ϕ(i)∈{0, …, 2L-1} represents the order of occurrence of state s in this sequence, and sign(.) denotes the sign of the argument.

Proof: Determining the rationality of a state sequence is equivalent to evaluating the solvability of the following equation system

$$\mathbf{Ah} = \tilde{\mathbf{x}} \tag{5}$$

where $\mathbf{h}$ denotes the channel coefficient vector, $\tilde{\mathbf{x}} = [x_{\phi(0)}, x_{\phi(1)}, \ldots, x_{\phi(2^L-1)}]^T$, and $\mathbf{A} = [a_0^T, a_1^T, \ldots, a_{2^L-1}^T]^T$, where $\mathbf{a}_i$ is the modulated signal vector corresponding to $\mathbf{s}_i$. As is well known, this equation system is solvable if and only if

$$\mathrm{rank}(\mathbf{A}) = \mathrm{rank}([\mathbf{A}, \tilde{\mathbf{x}}]) \tag{6}$$

For clear presentation, let $\overline{i}$ denote 2^L-1-i. Note that

$$\mathbf{a}_i + \mathbf{a}_{\overline{i}} = \mathbf{0}_{1\times L} \tag{7}$$

holds $\forall i \in \{0, 1, \ldots, 2^L\text{-}1\}$, wherein $\mathbf{0}_{1\times L}$ represents the $1\times L$ zero vector. Thus, when Eq. (6) is satisfied, we have

$$x_{\phi(i)} + x_{\phi(\overline{i})} = 0. \tag{8}$$

Making the substitution of $x_{\phi(i)} = -x_{\overline{\phi(i)}}$ in the above equation, we can easily arrive at Eq. (3). It should also be noted that $\forall l \in \{0, 1, \ldots, L\text{-}1\}$ and j, $k \in \Omega_l$,

$$\mathbf{a}_j + \mathbf{a}_{2^l+j} = \mathbf{a}_k - \mathbf{a}_{2^l+k}. \tag{9}$$

Accordingly, we have

$$x_{\phi(i)} - x_{\phi(2^l+j)} = x_{\phi(k)} - x_{\phi(2^l+k)}. \tag{10}$$

Eq. (4) then follows.

Let P≡[p_{ij}], i, j=0, 1, …, 2^L-1, be the transition matrix of ordered state sequence [$\mathbf{s}_k$], k = 0, 1, …, 2^L-1, i.e., p_{ij} is equal to 1 if the transition from $\mathbf{s}_i$ to $\mathbf{s}_j$, denoted as $\mathbf{s}_i \rightarrow \mathbf{s}_j$, is possible and 0 otherwise. Clearly, $\mathbf{s}_i \rightarrow \mathbf{s}_j$ occurs only if the earliest (rightmost) L-1 bits of $\mathbf{s}_j$ reproduces the latest (leftmost) L-1 bits of $\mathbf{s}_i$, i.e.,

$$\mathbf{s}_i \rightarrow \mathbf{s}_j \quad \text{if } j = (i >> 1) \text{ or } 2^{L-1} + (i >> 1) \tag{11}$$

where $i>>k$ denotes the right shift of i by k bits. Therefore, $\mathbf{P}$ is solely determined by L, the number of resolvable paths. Hereinafter, $\mathbf{P}$ will be referred to as the elementary transition matrix. Let $\mathbf{Q}$≡ [q_{ij}], be the origin transition matrix, i.e., q_{ij} is equal to 1 if the transition from x_i to x_j is possible and 0 otherwise. Since the origin transition matrix can be interpreted as the transition matrix of permuted state sequence [$\mathbf{s}(x_k)$], where $\mathbf{s}(x_k)$ denotes the state associated with the kth origin x_k, we can have the following theorem:

Theorem 2. Given $\mathbf{P}$ and $\mathbf{Q}$, the elementary state and the origin transition matrices of an arbitrary BPSK system, there exists at least one orthogonal, symmetric and centrosymmetric transform matrix $\mathbf{T}$ that satisfies

$$\mathbf{Q} = \mathbf{TPT}. \tag{12}$$

Proof: It is well known that a permutation like [$\mathbf{s}(x_k)$] can be derived from ordered state sequence [$\mathbf{s}_k$] through a series of element exchanges. In addition, the exchange of the ith and jth elements of a state sequence is equivalent to the exchanges of the ith and jth rows and columns of the transition matrix. Thus, $\mathbf{Q}$ can be obtained from $\mathbf{P}$ as follows

$$\mathbf{Q} = \mathbf{E}_l \mathbf{E}_{l-1} \ldots \mathbf{E}_1 \; \mathbf{P} \; \mathbf{E}_1 \ldots \mathbf{E}_{l-1} \; \mathbf{E}_l = \mathbf{TPT}. \tag{13}$$

wherein, $\mathbf{E}_k$, $1 \leqslant k \leqslant l$, denotes row or column exchange. Clearly, transform matrix $\mathbf{T}$ is orthogonal and symmetric. From Theorem 1, we know that state sequence $[\mathbf{s}(x_k)]$ is rational only if $\mathbf{s}(x_i)$ and $\mathbf{s}_{x_{\bar{i}}}$ are bitwise opposite for arbitrary $i \in \{0, 1, \ldots, 2^L\text{-}1\}$, i.e.,

$$\mathbf{s}(x_i) \oplus \mathbf{s}(x_{\bar{i}}) = \mathbf{s}_{2^L-1}$$

where $\oplus$ denotes exclusive-or. Hence, the (i, j) row or column exchange, if is involved in Eq. (13), must be succeeded by an $(\bar{i}, \bar{j})$ row or column exchange. The centrosymmetry of $\mathbf{T}$ then follows.

From Theorem 2 and the definition of origin transition matrix, it can be easily derived that the origin transition matrix of a BPSK system with L transmission paths has the following properties.

Property 1. The rank of the origin transition matrix is $2^{L\text{-}1}$, and each row and column has a duplicate in the matrix.

Property 2. The origin transition matrix is centrosymmetric, that is, $q_{ij} = q_{\bar{i}\bar{j}}$ holds for arbitrary $i, j \in \{0, 1, \ldots, 2^L\text{-}1\}$.

Property 3. There are two nonzero elements in each row and column. The L-bit state sequences associated with them are differentiated by the leftmost bit in the row direction and the rightmost in the column direction, i.e.,

$$\mathbf{s}(x_j) \oplus \mathbf{s}(x_k) = \mathbf{s}_{2^L-1}, \quad \text{if } q_{ij} = q_{ik} = 1;$$

$$\mathbf{s}(x_i) \oplus \mathbf{s}(x_k) = \mathbf{s}_1, \quad \text{if } q_{ij} = q_{kj} = 1.$$

Property 4. There are two nonzero elements on the main diagonal, corresponding to state $\mathbf{s}_0$ and $\mathbf{s}_{2^L-1}$ respectively.

It should be aware that the mapping from state sequences to state transition matrices is non-injective. For example, $[\mathbf{s}_0, \mathbf{s}_1, \mathbf{s}_2, \mathbf{s}_3]$ and $[\mathbf{s}_3, \mathbf{s}_2, \mathbf{s}_1, \mathbf{s}_0]$ have the same transition matrix.

Theorem 3. Two state sequences of equal length 2^L have the same state transition matrix if and only if for arbitrary i $\in \{0, 1, \ldots, 2^L\text{-}1\}$

$$\mathbf{s}_i^{(1)} \oplus \mathbf{s}_i^{(2)} = \mathbf{s}_{2^L-1}, \tag{14}$$

where $\mathbf{s}_i^{(1)}$ and $\mathbf{s}_i^{(2)}$ represent respectively the ith elements of the first and the second state sequences.

Proof: The sufficiency of Eq. (14) follows immediately from the fact that if transition $\mathbf{s}_i \rightarrow \mathbf{s}_j$ is possible, transition $\mathbf{s}_{\bar{i}} \oplus \mathbf{s}_{\bar{j}}$ is also possible.

To prove the necessity of Eq. (14), let us assume $\mathbf{s}_{\phi_1(k)}^{(1)} \oplus \mathbf{s}_k$, $k = 0, \ldots, 2^L\text{-}1$, where $\phi_1(k)$ is the order of occurrence of $\mathbf{s}_k$ in state sequence $[\mathbf{s}_k^{(1)}]$. It follows from Property 4 that $\mathbf{s}_{\phi_1(0)}^{(2)}$ is either $\mathbf{s}_0$ or $\mathbf{s}_{2^L-1}$.

Consider the latter case first. Since $\mathbf{s}_{2^L-1}$ can only be transmitted to, other than itself, $\mathbf{s}_{2^{L-1}-1}$, we arrive at $\mathbf{s}_{\phi_1(2^{L-1})}^{(2)} = \mathbf{s}_{2^{L-1}-1}$. Furthermore, as shown in Eq. (11), the transmission from $\mathbf{s}_{2^{L-1}-1}$ must end at $\mathbf{s}_{2^{L-2}-1}$ or $\mathbf{s}_{2^{L-1}+2^{L-2}-1}$ It then follows from Theorem 1 that $\mathbf{s}_{\phi_1(2^{L-2})}^{(2)} = \mathbf{s}_{2^{L-2}-1}$ and $\mathbf{s}_{\phi_1(2^{L-1}+2^{L-2})}^{(2)} = \mathbf{s}_{2^{L-1}+2^{L-2}-1}$. Repeating this procedure sequentially leads to the conclusion that Eq. (14) holds for arbitrary $i \in \{0, 1, \ldots, 2^L\text{-}1\}$. In the former case of $\mathbf{s}^{(2)}_{\phi_1(0)} = \mathbf{s}_{2^L-1}$, it can be easily verified that a duplicate of $[\mathbf{s}^{(1)}_k]$ is obtained in the end.

This study is concerned with rational state sequences. By combining Theorem 1 and 3, we can arrive at the following corollary.

Corrolary 1. Two rational state sequences of equal length have the same transition matrix if and only if they are inverse to each other.

In accordance with Corrolary 1, transform matrix **T** that satisfies Eq. (12) is not unique. This problem is considered in the following theorem.

Theorem 4. Given the elementary state and the origin transition matrices of a BPSK system, **P** and **Q** respectively, two qualified transform matrices can be found and they are related to each other as follows

$$\mathbf{T}_1\mathbf{T}_2 = \tilde{\mathbf{I}} \tag{15}$$

where $\mathbf{T}_1$ and $\mathbf{T}_2$ denote the qualified transform matrices, and $\tilde{\mathbf{I}}$ is a matrix with ‘1’ on the secondary diagonal and ‘0’ elsewhere.

Proof: It follows from Theorem 2 that

$$\mathbf{T}_1\mathbf{T}_2\mathbf{P}\mathbf{T}_2\mathbf{T}_1 = \mathbf{P}.$$

This suggests that, through a series of element exchanges associated with transform matrix $\mathrm{T}_1\mathrm{T}_2$, a new state sequence is obtained and it has the same state transition matrix with $[\mathbf{s}_i]$. From Corollary 1, we know that this sequence is the inverse of $[\mathbf{s}_i]$. The assertion follows.

As is shown above, the information contained in the origin transition matrix is by itself insufficient to guarantee the uniqueness of the configuration of the state-origin mapping system. Fortunately, this kind of ambiguity can be overcome easily with the knowledge of an arbitrary state-origin correspondence.

Theorem 5. Given an arbitrary state-origin correspondence, the state-origin mapping system can be uniquely determined by the origin transition matrix.

Proof: It can be seen from Corollary 1 that, given an origin transition matrix, two rational state sequences can be found to match it, and their elements are inverse in order to each other. It follows immediately that the two sequences are distinct at each position. Therefore, once an arbitrary origin is identified as the correspondence of a specific state, the two sequences can be distinguished and the state-origin mapping system is uniquely determined.

4. Conclusions

In this paper, a theoretical analysis of cluster-based channel identification is present. We show that, although an identification approach solely based on the origin transition matrix may result in some degree of channel ambiguity, the problem can be easily overcome with a simple training process. In fact, in many cases, the training process can be totally avoided (this issue will be discussed in another paper).

The theorems and the conclusions derived in this paper are also helpful for the exploration for new solutions to cluster-based channel identification.

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